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ЗОНЫ ПОСТОЯННЫХ МИНИМАЛЬНЫХ ЗНАЧЕНИЙ И ЗОНЫ ФЛУКТУАЦИИ ФУНКЦИОНАЛА ЭНЕРГИИ В ПРОМЕЖУТОЧНОМ РЕЖИМЕ ИНДУКЦИОННОГО НАГРЕВА ОБСАДНОЙ КОЛОННЫ НЕФТЯНОЙ СКВАЖИНЫ

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На основе полученных теоретических результатов [1, 2] проведены численные эксперименты в среде Borland Delphi 7 при различных значениях штрафных параметров *β, γ*, входящих в минимизируемый функционал, исследованы закономерности влияния этих параметров на его величину. Выявлены зоны постоянства значений функционала и зоны флуктуаций.

Ключевые слова: индукционный нагрев; обсадная колонна; нефтяная скважина; схема Кранка–Николсона; метод максимума Понтрягина; промежуточный режим нагрева.

FIELDS OF CONSTANT MINIMUM VALUES AND FIELDS OF FLUCTUATIONS OF THE ENERGY FUNCTIONAL IN THE INTERMEDIATE MODE OF THE INDUCTION HEATING OIL WELL CASING PIPE

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On the bases of the obtained theoretical results [1], [2] the numeric experiments were conducted in the Borland Delphi 7 environment, within different values of the penalty parameters *β, γ*, being a part of the minimized functionality, influence common factors of these parameters on its quantity are studied. Constancy zones of functionality value and fluctuations zones were explored.

Keywords: induction heating; casing; oil well; scheme of Krank-Nicolson; Pontryagin maximum method; intermediate heating mode.

The statement of the problem. Controlled process of the induction heating during transitory cycling with distributed energy resources in the $Q=\{0<t<t_1, 0<r<R\}$ system is described by heat-transfer equation

$$\frac{\partial T}{\partial t}=a\frac{\partial^2 T}{\partial r^2}+\frac{a}{r}\frac{\partial T}{\partial r}+\frac{a}{\lambda}V(t,r), \quad (1)$$

with initial-value and boundary conditions

$$T(0,r)=\varphi_1(r), \qquad \frac{\partial T(t,0)}{\partial r}=0, \qquad \left.\frac{\partial T(t,r)}{\partial r}\right|_{r=R}=h\left[T_R-T(t,R)\right], \quad (2)$$

$\frac{a}{\lambda}V(t,r)=w(r)u(t),\ w(r)$ – given function in the intermediate mode [2].

It is necessary to find among all alternative control circuits $0\le u(t)\le u_{\max}$ ***a control circuit*** $u_0(t)$***, which minimizes energy functional with an appropriate solution*** $T^0(t,r)$ ***of boundary value problem (1)– (2)*** $J[u]=2\pi l\int_0^R\int_0^{t_1}V(t,r)rdtdr$ and under $t=t_1$ ***is conducted*** $T(t_1,r)=\varphi_2(r)$**.** The problem of energy functional minimization is substituted by a problem of the functional form minimization [2]

$$F[u,\beta,\gamma,c]=\beta\left\{\gamma\int_0^{t_1}u(t)dt+\int_0^{t_1}[u(t)-c]^2dt\right\}+\int_0^R r[T(t_1,r)-\varphi_2(r)]^2dr. \quad (3)$$

As a result of substitution $v(t)=u(t)-c+\frac{\gamma}{2}$ and functional transformation (3) a quadratic functional is developed

$$F\left[v,\beta,\gamma,c\right]=\beta\int_0^{t_1} v^2(t)dt+\int_0^R r\left[T\left(t_1,r\right)-\varphi_2(r)\right]^2 dr-\frac{\gamma\beta t_1\left(\gamma-4c\right)}{4}. \quad (4)$$

The solutions of the basic and adjoint problems. Solving the problem (1)–(2) one should apply Pontryagin maximum method for distributed parameter systems [1] in the induction heating optimization [2] and integro-interpolated method [3].

The following problem is:

$$\frac{\partial T^{(k)}\left(t,r\right)}{\partial t}=\frac{a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T^{(k)}}{\partial r}\right)+u^{(k-1)}(t)w(r), \quad (5)$$

$$T^{(k)}\left(0,r\right)=\varphi_1(r),\quad \left.\frac{\partial T^{(k)}(t,r)}{\partial r}\right|_{r=R}=h\left[T_R-T^{(k)}\left(t,R\right)\right]. \quad (6)$$

The transformations with [4] quadrature are:

$$\frac{1}{\tau}\int_{t_{n-1}}^{t_n} T^{(k)}\left(t,r\right)dt\approx\theta T^{(k)}\left(t_n,r\right)+\left(1-\theta\right)T^{(k)}\left(t_{n-1},r\right),$$

leading to the boundary value problem for the ordinary differential expression

$$\frac{a}{r}\left(rU'(r)\right)'-bU(r)=f(r); \quad (7)$$

$$U'(0)=0;\quad U'(R)=h_z\left[T_R-U(R)\right], \quad (8)$$

where the following nomenclatures are inserted:

$$U(r)\equiv\theta T^n(r)+(1-\theta)T^{n-1}(r),\ T^n(r)\equiv T^{(k)}(t_n,r),\ b=\frac{1}{\theta\tau};\ f(r)=-\frac{T^{n-1}(r)}{\theta\tau}-w(r)u_{n-1}.$$

Creating Krank-Nicolson difference scheme for the problem (5) – (6), discretized equation system can be developed

$$\begin{cases} a\frac{U_2^h-U_1^h}{h}=\frac{h}{4}\left(f_1+bU_1^h\right); \\ a\frac{r_{i+1}}{r_i}S\left(\frac{r_i}{r_{i+1}}\right)\frac{U_{i+1}^h-U_i^h}{h}-aS\left(\frac{r_{i-1}}{r_i}\right)\frac{U_i^h-U_{i-1}^h}{h}=\frac{1}{2}\left(f_i+bU_i\right)\left[\frac{r_{i+1}}{r_i}S\left(\frac{r_i}{r_{i+1}}\right)r_{i+1/2}-S\left(\frac{r_{i-1}}{r_i}\right)r_{i-1/2}\right]; \\ i=\overline{1,N-1} \\ -aS\left(\frac{r_{N-1}}{r_N}\right)\frac{U_N^h-U_{N-1}^h}{h}+ah_zT_R-ah_zU_N^h=\frac{1}{2}\left(f_N+bU_N\right)\left[-S\left(\frac{r_{N-1}}{r_N}\right)r_{N-\frac{1}{2}}+r_N\right]. \end{cases} \quad (9)$$

In such a manner, (5)–(6) is approximated by the task (9), its solution is developed with the help of the marching method, after which the solution of the initial problem (5)–(6) is defined according to the following equation:

$$T_i^n=\frac{1}{\theta}U_i^h-\left(\frac{1}{\theta}-1\right)T_i^{n-1},\quad i=1,2,...,N. \quad (10)$$

Adjoint problem is solved.

$$\frac{\partial\psi^{(k)}(t,r)}{\partial t}+a\frac{\partial^2\psi^{(k)}(t,r)}{\partial r^2}+\frac{a}{r}\frac{\partial\psi^{(k)}(t,r)}{\partial r}=0,$$

$$\psi^{(k)}(t_1,r)=-2[T^{(k)}(t_1,r)-\varphi_2(r)],\ \left[\frac{\partial\psi^{(k)}(t,r)}{\partial r}+h_z\psi^{(k)}(t,r)\right]_{r=R}=0.$$

This problem is solved with the help of the same method as basic one [4]. In the light of the identification $\Phi(r) \equiv (1-\theta)\psi^n(r) + \theta\psi^{n-1}(r)$ it is received the next boundary value problem for ordinary differential expression:

$$\frac{a}{r}\left(r\Phi'(r)\right)' - b\Phi(r) = f(r); \tag{11}$$

$$\Phi'(0) = 0; \quad \Phi'(R) = -h_z\Phi(R). \tag{12}$$

To this problem, scheme (9) is applied, where $T_R = 0$, value Φ_i^h is found, then iterative value for the solution of adjoint problem (10) is determined from the formula:

$$\psi_i^{n-1} = \frac{1}{\theta}\Phi_i^h - \left(\frac{1}{\theta} - 1\right)\psi_i^n, \quad i = 1, 2, ..., N. \tag{13}$$

After finding of an approximate solution of the adjoint problem and usage of the quadrature formula, a optimal control action on each iterative step according to the formula is found:

$$u^{(k)}(t_n) = \frac{1}{2\beta}\frac{h}{2}\sum_{i=1}^{I-1}\left\{r_i\psi_i^n w(r_i) + r_{i+1}\psi_{i+1}^n w(r_{i+1})\right\} + c - \frac{\gamma}{2}.$$

Then the transition to the next iteration is done, this process continues until the quantity of iterations does not exceed defined quantity, or when of control value change will be insignificant.

Numeric experiments analysis in the intermediate heating mode. Numeric implementation is realized in circumference Borland Delphi 7 environment. The program allows realizing the choice of the penalty parameters. Superficial diagrams were executed in Excel-2007.

Distribution function of internal heat sources in intermediate heating mode is given by [2]. At numerical calculations it was used the results of [5].

In intermediate heating mode initial temperature distribution is given by the function $\varphi_1(r) = 270 + 480r/R$, resulting at the end of cold mode [6]. Last temperature distribution is given by the function $\varphi_2(r) = 670 + 180r/R$.

It is examined and analyzed dependences of minimized function from penalty parameters γ, β.

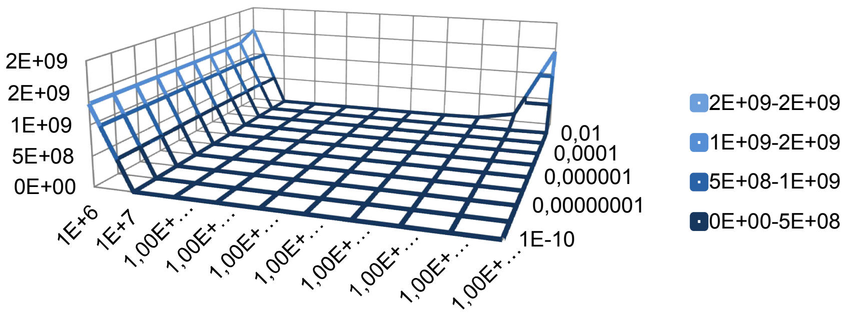
Heating time $t = 60$, specific power varies from $c = 0{,}0001$ till $c = 0{,}1$. Calculations showed, that if $\beta \in [10; 1E+5]$ functional takes on the negative values, therefore the further studies were done at $\beta \geq 1E+6$.

Chosen value dependence calculations results of functional F from parameters γ, β if $c = 0{,}001$ are listed in table 1 and it was illustrated by the superficial diagram (pic. 1).

Table 1 – Dependence of functional F on parameters γ, β at $\beta \in [1E+6, 1E+15]$

$\gamma \setminus \beta$	1E+6	1E+8	1E+9	1E+10	1E+11	1E+12	1E+13	1E+14	1E+15
1E-1	1,52E+9	110,2	1403,7	14363	1,4E+5	1,4E+6	1,4E+7	1,4E+8	1,4E+9
1E-2	1,35E+9	7,45	0,16	81,11	891,11	8991,	9E+04	9E+05	90E+6
1E-3	1,33E+9	6,93	7,70	11,79	52,30	457,30	4507,3	45007,3	450007
1E-4	1,33E+9	6,75	7,17	7,73	13,00	65,65	592,15	5857,15	58507
1E-5	1,329E+9	6,73	7,10	7,19	7,73	13,12	66,99	605,64	5992
1E-6	1,329E+9	6,73	7,09	7,14	7,19	7,74	13,13	67,12	606
1E-7	1,329E+9	6,73	7,09	7,13	7,14	7,20	7,74	13,13	67,13
1E-8	1,329E+9	6,73	7,09	7,13	7,14	7,14	7,2	7,74	13,14
1E-9	1,329E+9	6,73	7,09	7,13	7,13	7,14	7,14	7,2	7,74
1E-10	1,329E+9	6,73	7,09	7,13	7,13	7,14	7,14	7,14	7,2

From the table 1 it is possible to see that with reduction of parameters γ in the range of $\gamma \in [1E-4; 1E-1]$, the functional values are considerably reduced, the further reducing of this parameter $\gamma \in [1E-10; 1E-5]$ does not influence on functional amt. The increasing of β from $1E+6$ till $1E+7$ leads to a sharp reduction of the functional. The further increasing of β leads to the functional rising.



Picture 1 – Superficial diagram $F(\beta,\gamma)$ if $c=0,001$

The field of relatively constant minimum values of the functional – the area of prosperous functional values are accrued from analysis. Minimum values of the functional is observed by $\beta\in[1E+6;1E+9]$, $\gamma\in[1E-8;1E-5]$.

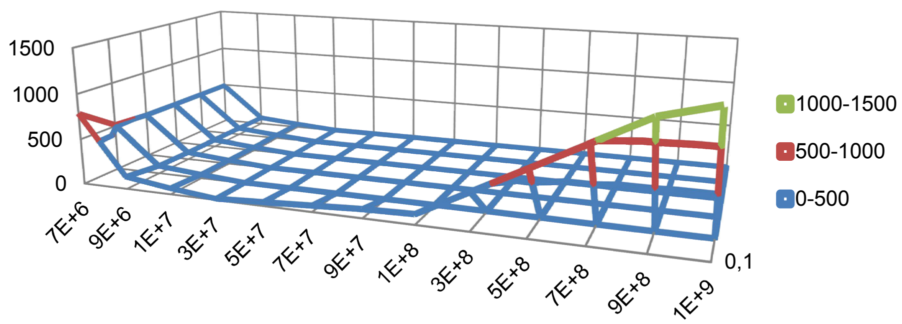
Field of constant minimum values of the functional further gives the opportunity to correct the received results due to the temperature distribution across the cross section of the end of the heating cylinder.

In the table 2 is given the increasing of iterations on β in recommended change range of value β for correction of minimum values of the functional.

Table 2 – Dependence of functional F on parameters γ, β At $\beta\in[7E+6,1E+9]$

$\gamma\backslash\beta$	7E+6	1E+7	3E+7	5E+7	7E+7	1E+8	3E+8	5E+8	7E+8	9E+8	1E+9
1E-1	783,3	67,1	14,42	40,6	68,1	110,2	396	684,1	971,9	1260	1404
1E-2	514,5	43,3	7,11	7,43	7,51	7,45	6	4,28	2,51	0,73	0,16
1E-3	490,30	40,48	6,15	6,55	6,75	6,93	7,29	7,43	7,55	7,65	7,70
1E-4	487,91	40,19	6,02	6,40	6,59	6,75	7,03	7,09	7,13	7,16	7,17
1E-5	487,67	40,17	6,01	6,39	6,58	6,73	7	7,05	7,08	7,09	7,10
1E-6	487,65	40,16	6,01	6,38	6,58	6,73	6,99	7,05	7,07	7,09	7,09

On superficial diagram (pic. 2) by increasing of iterations till β the fluctuation areas are appeared by $\gamma=0,1$, $\beta\in[7E+6;1E+9]$ and by $\beta=7E+6$, $\gamma\in[1E-6;1E-1]$, except this fact the areas in which functional accepts the least value: the first area – $\beta=3E+7$, $\gamma\in[1E-6;1E-3]$; the second area – $\beta\in[3E+8;1E+9]$, $\gamma=1E-2$. The controlling accepts the negative value by $\beta\in[3E+8;1E+9]$, $\gamma=1E-2$.



Picture 2 – Superficial diagram $F(\beta,\gamma)$ by $c=0,001$

Conclusion: for $c \in [0,001; 0,1]$ the area of minimal values of functional is observed by $\gamma \in [1E-6; 1E-4]$, $\beta \in [1E+7; 1E+8]$.

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