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ИНВЕРСНО-ЗАМКНУТЫЕ КОЛЬЦА НА РАВНОМЕРНЫХ ПРОСТРАНСТВАХ

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Получены равномерные аналоги теорем Стоуна и Гельфанда – Колмогорова о характеризации тихоновских пространств посредством колец всех (ограниченных) непрерывных функций.

Ключевые слова: кольцо; идеал; максимальный идеал; компактификация; *coz*-тонкое пространство.

INVERSION-CLOSED RINGS ON UNIFORM SPACES

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The uniform analogues of Stone and Gelfand-Kolmogoroff theorems on characterization of Tychonoff spaces by means of the rings of all (bounded) continuous functions have been proved.

Keywords: ring, ideal; maximal ideal; compactification; *coz*-fine space.

Introduction

Given paper is an organic continuation of the previous paper "On β-like compactification of the uniform spaces", so all denotations and references from the first paper are similar.

Main results

Let $\mathcal{M}\left(C_u^*\left(\beta_u X\right)\right)$ ($\mathcal{M}\left(C_u^*\left(X\right)\right)$, $\mathcal{M}\left(C_u\left(X\right)\right)$) be a collection of all maximal ideals of commutative ring with unity $C_u^*\left(\beta_u X\right)$ ($C_u^*\left(X\right), C_u\left(X\right)$) with the Stone topology [1]. For a compactification $\beta_u X$ the Stone Theorem is formulated as it follows down:

Theorem 2.1. [1]. *For a compactification $\beta_u X$ the maximal ideals of $C^*\left(\beta_u X\right)$ are in the one-to-one correspondence with the points of $\beta_u X$ and are given $I_p^* = \left\{f \in C^*\left(\beta_u X\right) : f\left(p\right) = 0\right\}$ for p is a point of $\beta_u X$.*

Let $\mathcal{M}\left(C^*\left(\beta_u X\right)\right)$ be denote as $\mathcal{M}^*\left(\beta_u X\right)$, $\mathcal{M}\left(C_u^*\left(\beta_u X\right)\right)$ as $\mathcal{M}^*\left(uX\right)$ and $\mathcal{M}\left(C_u\left(X\right)\right)$ as $\mathcal{M}\left(uX\right)$.

Theorem 2.2. [1]. *A compact space $\beta_u X$ is homeomorphic to the maximal ideals space $\mathcal{M}^*\left(\beta_u X\right)$.*

The following corollary is immediately implicated from the observation that the ring $C_u^*\left(X\right)$ is isomorphic to $C^*\left(\beta_u X\right)$.

Corollary 2.3. *A compactification $\beta_u X$ is homeomorphic to the maximal ideals space $\mathcal{M}^*\left(uX\right)$.*

If $C_u^*\left(X\right)$ and $C_v^*\left(Y\right)$ are isomorphic for a uniform spaces uX and vY, then $\mathcal{M}^*\left(uX\right)$ and $\mathcal{M}^*\left(vY\right)$ are homeomorphic.

Corollary 2.4. *For a uniform spaces uX and vY its compactifications $\beta_u X$ and $\beta_v Y$ are homeomorphic if and only if $C_u^*\left(X\right)$ and $C_v^*\left(Y\right)$ are isomorphic.*

From the Theorems 2.4, 2.8., 2.16 of the previous paper the next result holds.

Corollary 2.5. *Let uX and vY be the first-countable uniform spaces. Then uX is $coz-$homeomorphic to vX if and only if $C_u^*\left(X\right)$ is isomorphic to $C_v^*\left(Y\right)$.*

Corollary 2.6. *Let uX and vY be the first-countable $coz-fine$ uniform spaces. Then uX is uniformly homeomorphic to vY if and only if $C_u^*\left(X\right)$ is isomorphic to $C_v^*\left(Y\right)$.*

Corollary 2.7. *Let uX and vY be the first-countable $coz-$fine uniform spaces. Then a completion $\mu_u X$ is uniformly homeomorphic to a completion $\mu_v Y$ if and only if $C_u^*(X)$ is isomorphic to $C_v^*(Y)$.*

For $coz-$ fine uniform spaces uX and vY $C_u^*(X)=U^*(uX)$ and $C_v^*(Y)=U^*(vY)$ [2], [3].

Corollary 2.8. *Let uX and vY be the first-countable $coz-$fine uniform spaces. Then a completion $\mu_u X$ is uniformly homeomorphic to a completion $\mu_v Y$ if and only if $U^*(uX)$ is isomorphic to $U^*(vY)$.*

Corollary 2.9. *Let uX and vY be a complete the first-countable $coz-$fine uniform spaces. Then uX is uniformly homeomorphic to vY if and only if $U^*(uX)$ is isomorphic to $U^*(vY)$.*

For the $C_u^*(X)$ we can utilize the ring isomorphism $f \mapsto \beta_u f$ of $C_u^*(X)$ and $C^*(\beta_u X)$ to characterize the maximal ideals of $C_u^*(X)$ in terms of $\beta_u X$.

Theorem 2.10. *The maximal ideals of $C_u^*(X)$ are in the one-to-one correspondence with the points of $\beta_u X$ and are given $I_p^* = \{f \in C^*(X) : \beta_u f(p)=0\}$ for p is a point of $\beta_u X$.*

For the relate z_u-filters and z_u-ultrafilters to the ring $C_u(X)$ consider the function $\mathcal{Z}: C_u(X) \to Z_u$ ($\mathcal{Z}(f)=f^{-1}(0) \in Z_u$ for $f \in C_u(X)$). The following result is analogue [4, 2.3] and shows that the image of an ideal (maximal ideal) under $\mathcal{Z}$ is a z_u-filter (z_u-ultrafilter) and that the preimage of a z_u-filter (z_u- ultrafilter) is an ideal (maximal ideal).

Proposition 2.11. *(a) If I is a proper (maximal) ideal in $C_u(X)$, then $\mathcal{Z}(I)=\{\mathcal{Z}(f) : f \in I\}$ is a z_u- filter (z_u-ultrafilter) on uX.*

(b) If $\mathcal{F}$ is a z_u-filter (z_u-ultrafilter) on uX, then $\mathcal{Z}^{-1}[\mathcal{F}]=\{f \in C_u(X) : \mathcal{Z}(f) \in \mathcal{F}\}$ is an (maximal) ideal in $C_u(X)$.

Theorem 2.12. *The maximal ideals of $C_u(X)$ are in the one-to-one correspondence with the points of $\beta_u X$ and are given $I_p = \left\{f \in C_u(X) : p \in [\mathcal{Z}(f)]_{\beta_u X}\right\}$ for p is a point of $\beta_u X$.*

Proof. It is analogically to the proof of Theorem 1.30 [5].

Corollary 2.13. *A compactification $\beta_u X$ is homeomorphic to the maximal ideals space $\mathcal{M}(uX)$.*

Proof. It is analogically to the proof of Corollary 1.31 [5].

If $C_u(X)$ and $C_v(Y)$ are isomorphic for a uniform spaces uX and vY, then $\mathcal{M}(uX)$ and $\mathcal{M}(vY)$ are homeomorphic. Q.E.D.

Corollary 2.14. *For a uniform spaces uX and vY its compactifications $\beta_u X$ and $\beta_v Y$ are homeomorphic to each other if and only if $\mathcal{M}(uX)$ and $\mathcal{M}(vY)$ are isomorphic to each other.*

Corollary 2.15. *Let uX and vY be the first-countable uniform spaces. Then uX is $coz-$homeomorphic to vX if and only if $C_u(X)$ is isomorphic to $C_v(Y)$.*

Corollary 2.16. *Let uX and vY be the first-countable $coz-$fine uniform spaces. Then uX is uniformly homeomorphic to vY if and only if $C_u(X)$ is isomorphic to $C_v(Y)$.*

Corollary 2.17. *Let uX and vY be the first-countable $coz-$fine uniform spaces. Then a completion $\mu_u X$ is uniformly homeomorphic to a completion $\mu_v Y$ if and only if $C_u(X)$ is isomorphic to $C_v(Y)$.*

For $coz-$ fine uniform spaces uX and vY $C_u(X)=U(uX)$ and $C_v(Y)=U(vY)$ [2], [3].

Corollary 2.18. *Let uX and vY be the first-countable $coz-$fine uniform spaces. Then a completion $\mu_u X$ is uniformly homeomorphic to a completion $\mu_v Y$ if and only if $U(uX)$ is isomorphic to $U(vY)$.*

Corollary 2.19. *Let* uX *and* vY *be a complete the first-countable* $coz-$*fine uniform spaces. Then* uX *is uniformly homeomorphic to* vY *if and only if* $U(uX)$ *is isomorphic to* $U(vY)$.

Remark 2.20. Note, that Theorem 2.10 is a uniform analogue of Stone Theorem [1] and Theorem 2.12 is a uniform analogue of Gelfand – Kolmogoroff Theorem [6].

References

1. *Stone M*. Applications of the theory of Boolean rings to general topology / M. Stone // Trans. Amer. Math. Soc. 41 (1937). P. 375–481
2. *Frolik Z*. A note on metric-fine spaces / Z. Frolik // Proceeding of the American Mathematical Society. 1974. V. 46. № 1. P. 111–119.
3. *Frolik Z*. Four functor into paved spaces / Z. Frolik // In seminar uniform spaces 1973, 4. Matematický ústav ČSAV. Praha, 1975. P. 27–72
4. *Gillman L*. Rings of continuous functions / L. Gillman, M. Jerison // The Univ. Series in Higher Math. Van Nostrand, Princeton, N. J., 1960. 303 p.
5. *Walker R*. The Stone-Čech compactification / R. Walker // Springer-Verlag. New York, Berlin, 1974. 333 p.
6. *Gelfand J*. On rings of continuous function on topological spaces / J. Gelfand, A. Kolmogoroff // Dokl. Akad. Nauk SSSR 22, 1939. P. 11–15. (in Russian).